EXAMINATION 2004/05

SIGNAL PROCESSING

Duration: 120 mins

Answer THREE questions out of SIX with at least one from each section.

Calculators with text storage may be taken into the examination room but the text storage should NOT be used.

An approximate marking scheme is indicated.

Coursework contribution: 10%.

Section A, Question 1

a) State the real form of the Fourier series of a signal $x(t)$ of period T. Hence derive the complex form

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$$

where

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-jn\omega_0 t}\, dt,\ n \neq 0$$

where all symbols have their normal meanings. (*8 marks*)

b) A signal of period T is defined by

$$x(t) = \begin{cases} 0, & -\frac{T}{2} \leq t < 0 \\ 2, & 0 < t \leq \frac{T}{2} \end{cases}$$

Show that in the complex form of the Fourier series for this signal that $c_0 = 1$, and for $n \neq 0$

$$c_n = \begin{cases} -\frac{2j}{n\pi}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

Hence sketch the amplitude and power spectrums of this signal. (*12 marks*)

c) The signal $x(t)$ is only partially known in the sense that

$$x(t) = \begin{cases} 0, & t < 0 \\ e^{-at}, & 0 \leq t \leq T \end{cases}$$

The values for $t > T$ are **unknown** but it is also known that the Fourier transform $X(\omega)$ of $x(t)$ is given by

$$\begin{aligned} X(\omega) &= |X(\omega)|e^{j\theta(\omega)} \\ \theta(\omega) &= -\omega T \end{aligned}$$

Use properties of the Fourier transform to find $x(t)$ for all time. Hence sketch the amplitude and power spectrums of this signal. (*10 marks*)

Section A, Question 2

a) A particular application requires a linear discrete time system where, for each n, the output $y[n]$ is the average of the inputs at $n, n-1, n-2$. Determine the difference equation describing the input output response for this system. Then determine the frequency response $H(z)$ for this system and the associated pole-zero plot. (*9 marks*)

b) An alternative design would replace $H(z)$ by $G(z)$ which satisfies the difference equation

$$y[n] = \gamma y[n-1] + \beta x[n]$$

Under what condition will these two systems have **the same response** to a constant input. (*6 marks*)

c) A non-causal linear time invariant system has unit impulse response

$$h[n] = \begin{cases} -1, & n = -1 \\ 1, & n = 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find an explicit expression for the output $y[n]$ when the input is

$$x[n] = 1 + \sin\left(\frac{n\pi}{2}\right), \; n = 0, \pm1, \pm2, \ldots$$

Also find a causal system with the same frequency response and explain your reasoning. (*15 marks*)

Section A, Question 3

a) Write notes on the following for an analogue lowpass Butterworth filter with transfer function $H(s)$:

(i) passband edge
(ii) stopband edge
(iii) stopband attenuation

(*6 marks*)

b) Give an expression for $|H(j\omega)|^2$ for a lowpass Butterworth filter, passband edge ω_o, and order n. Determine the poles of $|H(j\omega)|^2$ in the ω plane and those of $H(s)$ in the s-plane. Show all relevant analysis.

(*9 marks*)

c) Design a highpass analogue Butterworth filter of order 3, with a passband edge at $1\ \mathrm{kHz}$. What is the magnitude response (in dB) of the filter at $500\ \mathrm{Hz}$? Comment on this result. What order filter n would give a rejection of $-60\ \mathrm{dB}$ at $500\ \mathrm{Hz}$. Give your filter solution as a ratio of real polynomials in the Laplace transform variable s. (*15 marks*)

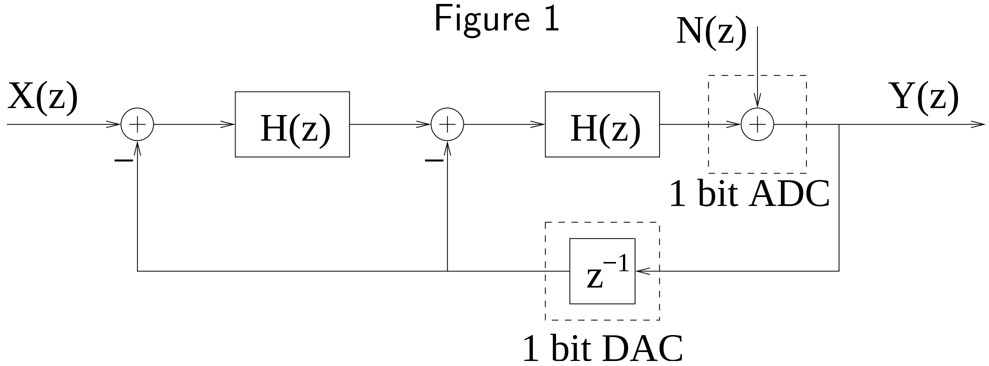
Section B, Question 4

a) State the purpose of preconditioning in analogue to digital conversion (ADC). (*3 marks*)

b) A quantiser with step size q causes quantisation noise with power $q^2/12$. Derive a formula for the SQNR as a function of of the number of bits, B, of the ADC. (*4 marks*)

c) Draw the block diagram of an oversampled ADC, and briefly show how oversampling can enhance the bit resolution. (*7 marks*)

Figure 1



d) Figure 1 provides the simplified discrete model of a 2nd order Σ-Δ converter with integrators $H(z) = 1/(1 - z^{-1})$. Identify the transfer functions $H_{\mathrm{X}}(z)$ and $H_{\mathrm{N}}(z)$ such that the output $Y(z)$ can be written as

$$Y(z) = H_{\mathrm{X}}(z)X(z) + H_{\mathrm{N}}(z)N(z) \quad . \tag{1}$$

(*6 marks*)

e) Derive and sketch the magnitude responses $|H_{\mathrm{X}}(e^{j\Omega})|$ and $|H_{\mathrm{N}}(e^{j\Omega})|$. (*5 marks*)

f) Basing your argumentation on oversampling, your results in (d), and appropriate postprocessing of $Y(z)$, describe how a Σ-Δ converter can achieve high bit resolution. (*5 marks*)

Section B, Question 5

a) An LTI system with input $x[n]$ and output $y[n]$ is described by the difference equation

$$y[n] = \sum_{k=0}^{N-1} \left(\frac{1}{2}\right)^k x[n-k] \qquad (2)$$

where N is arbitrary. Draw a block diagram of the system.

(*4 marks*)

b) Which of the following properties does the system in (a) fulfill? Justify your answer.

i) finite / infinite impulse response;

ii) causal / anti-causal / non-causal system;

iii) linear / non-linear phase;

iv) minimum / maximum / non-minimum phase.

(*4 marks*)

c) State the transfer function of the system in (2), $H(z) = Y(z)/X(z)$.

(*3 marks*)

d) Show that the filter $H(z)$ can also be represented as a recursive filter.

(*7 marks*)

e) Draw a flow graph of your system $H(z)$ in (d). For which order N would you prefer a recursive implementation? (*6 marks*)

f) Briefly discuss the trade-off between FIR and IIR filter with respect to

i) computational complexity;

ii) sensitivity to fixed point implementation;

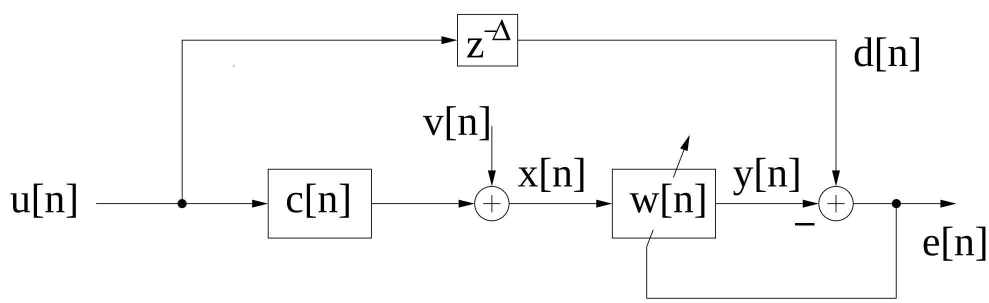
iii) linear / non-linear phase; (*6 marks*)

Section B, Question 6

a) Draw the block diagram of a generic adaptive filter and briefly describe its operation. (*4 marks*)

b) Consider the inverse system identification or equalisation setup in Figure 2. Given $c[n] \circ\!\!-\!\!\bullet\, C(z) = 1 + \frac{1}{2}z^{-1}$, $\Delta = 0$, and $v[n] = 0\ \forall n$, determine $w[n]$ as the inverse of $c[n]$. (*5 marks*)

Figure 2



c) With three coefficients in the adaptive filter, the covariance matrix of the noise-free input, $\mathbf{R}_{xx}$, has been estimated as

$$\mathbf{R}_{xx} = \mathbf{U} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0.01 \end{bmatrix} \mathbf{U}^{\mathrm{H}} \qquad (3)$$

with an appropriate unitary $\mathbf{U}$. State the Wiener-Hopf solution for the filter $w[n]$ as a function of $\mathbf{U}$ and the cross-correlation vector $\mathbf{p}$. In practice, would this solution be likely to differ from your solution in (b)? (*5 marks*)

d) Modify your Wiener-Hopf solution in (c) to accommodate channel noise $v[n] \in \mathcal{N}(0,1)$ independent of $u[n]$. How does this affect the inverse identication of $c[n]$? (*4 marks*)

e) Define the instantaneous squared error for the system in Figure 2 and derive the least mean squares (LMS) algorithm. (*6 marks*)

f) The unknown system is now given by $c[n] \circ\!\!-\!\!\bullet\, C(z) = \frac{1}{2} + z^{-1}$. For the noise-free case $v[n] = 0 \forall n$, show that only by permitting a delay $\Delta > 0$ in Figure 2, the LMS can converge to a solution. (*6 marks*)

END OF PAPER