Altruistic/Egocentric Optimization and Multiuser Transmitter/Receiver Optimization

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Motivation and Outline

- Introduction;
- Principles of Altruistic (A)-optimization and Egocentric (E)-optimization;
- Multiple-input multiple-output (MIMO) channel modelling;
- Equivalency between transmitter and receiver optimization;

Motivation and Outline (Continued)

- Design of multiuser transmitter (MUT) algorithms from multiuser detection (MUD) algorithms;
  - Matched-filtering;
  - Zero-forcing;
  - Minimum mean-square error;
  - Minimum variance distortionless response;
  - Minimum power distortionless response;
  - Maximum signal-to-interference-plus-noise ratio;
- Joint base-station MUD/MUT design;
- Performance examples;
- Further applications;

Introduction

- Transmitter preprocessing achieving multiuser transmission (MUT) is capable of suppressing the multiuser interference by carrying out signal processing at the transmitter side, if the transmitter employs the channel knowledge about the broadcast channels;
- With the aid of the MUT techniques, it is possible to implement low-complexity and high power-efficiency mobile terminals (MTs) for mobile broadcast channels;
- MUT is well suitable for cellular wireless communications systems using time-division duplex (TDD), since, in this case, the broadcast downlink channels can be estimated using their reciprocity with the multiple-access uplink channels;
Introduction (Continued)

- However, for the frequency-division duplex (FDD)-based wireless systems, feedback from the MTs to the BS transmitter may be necessary, since in FDD systems the uplink and downlink channels are not reciprocal.
- Multiuser detection was invented by Verdu in 1980s;
- There exist a wide range of high-efficiency MUD algorithms for various communications scenarios;
- Hence, if certain equivalency between MUT and MUD can be found, the well-built MUD will be significantly beneficial to the research of the less-studied MUT;

Altruistic-Optimization vs. Egocentric-Optimization

- **Egocentric (E)-Optimization**: The optimization is self-centred without considering the impact on the environment. In multiuser processing E-optimization aims at maximizing a given user’s performance without paying attention to its corresponding effect on the other users;
  ✓ If achieved, we say the solutions or systems are **E-optimum**.
- **Altruistic (A)-Optimization**: The optimization is altruistic, aiming at mitigating its impact on environment. In multiuser processing A-optimization motivates to minimize the interference of a given user on its colleague users, while maximizing the given user’s performance.
  ✓ If achieved, we say the solutions or systems are **A-optimum**.
- We say a multiuser communications system is **overall (O)-optimum**, if all the individual users are either E-optimum or A-optimum.

Multiuser MIMO Modelling - Assumptions

- A MIMO system having one base-station (BS) serving $K$ remote mobile terminals (MTs);
- The BS is mounted with $N$ antennas which are sufficiently separated, resulting that the uplink or downlink channels in the context of these antennas are independent;
- Each of the $K$ MTs employs one antenna for transmission and receiving;
- There are no cooperations among the MTs.
Multiuser MIMO Modelling - Uplink

When the uplink transmission is considered, the received $N$-length vector at the BS can be expressed as

$$y_U = Hx + n_U = \sum_{k=1}^{K} h_k x_k + n_U \quad (1)$$

where $x$ contains the $K$ symbols transmitted by the $K$ MTs to the BS, which is given by $x = [x_1, x_2, \ldots, x_K]^T$. It is assumed that $E[|x_k|^2] = 1$ and $x_k$ is an iid uniform random variable;

$H$ is the $(N \times K)$ MIMO channel transform matrix, which is given by

$$H = [h_1, h_2, \ldots, h_K] \quad (2)$$

where $h_k$ is the $k$th user’s spatial signature given by $h_k = [h_{k1}, h_{k2}, \ldots, h_{kN}]^T$, $k = 1, 2, \ldots, K$.

Multiuser MIMO Modelling - Uplink

When a linear detector is employed for detection of the uplink transmitted symbols, the decision variable vector for the $K$ transmitted symbols can be formed as

$$z_U = W^H y_U = W^H Hx + W^H n_U \quad (3)$$

where $W$ is a $(N \times K)$ matrix referred to as the receiver post-processing matrix, which can be denoted as

$$W = [w_1, w_2, \ldots, w_K] \quad (4)$$

$w_k, \ k = 1, 2, \ldots, K$ is the post-processing vector for detection of the $k$th uplink user.

Multiuser MIMO Modelling - Downlink

When the downlink transmission using linear transmitter preprocessing is considered, as shown in Fig.1, due to the reciprocity of the uplink and downlink MIMO channels, the received $K$-length vector by the $K$ remote MTs can be expressed as

$$y_D = H^T P x + n_D \quad (6)$$

where, in addition to $H$ and $x$, $P$ is the $(N \times K)$ transmitter preprocessing matrix, which is given by

$$P = [p_1, p_2, \ldots, p_K] \quad (7)$$

with $p_k$ being the vector for preprocessing $x_k$, $k = 1, \ldots, K$. 

Multiuser MIMO Modelling - Uplink

When the uplink detection is expressed in correspondence with each individual user, the decision variable for the $k$th uplink symbol $x_k$ is given by

$$y_{U,k} = w_k^H h_k x_k + \sum_{l \neq k} w_l^H h_l x_l + w_k^H n_U, \ k = 1, \ldots, K \quad (5)$$
Multiuser MIMO Modelling - Downlink

- When the downlink transmission is expressed in correspondence with each individual user, the decision variable for the $k$th downlink symbol $x_k$ is given by

$$y_{D,k} = h^T_k p_k x_k + \sum_{l \neq k} h^T_k p_l x_l + n_{D,k}, \quad k = 1, \ldots, K$$  

(8)

MUT vs. MUD - Observations

- Let $P = W^*$. Then, from (3) and (6), we have

$$z_U = W^H H x + W^H n_U$$
$$y_D = H^T W^* x = (W^H H)^T x + n_D$$  

(11)

- When ignoring the noise, we can see that both $z_U$ and $y_D$ have the same statistical properties due to the random properties of $x$;

- Hence they should yield the same error performance for both the uplink and downlink, when communicating over noise-less channels;

- Furthermore, any MUD scheme should be similarly efficient for MUT with regarding to the capability of MUI suppression.

Multiuser MIMO Modelling - Background Noise

- In (1) $n_U$ is the noise vector observed at the BS, which is assumed to be a Gaussian noise vector distributed with mean zero and a covariance matrix

$$E [n_U n_U^T] = \sigma^2_U I_N$$  

(9)

where $\sigma^2_U = 1/\text{SNR}_U$ and SNR$_U$ denotes the SNR of the uplink;

- In (5) or (8) $n_D$ is the noise vector observed by the MTs, which is also assumed to be a Gaussian noise vector distributed with mean zero and a covariance matrix

$$E [n_D n_D^T] = \sigma^2_D I_K$$  

(10)

where $\sigma^2_D = 1/\text{SNR}_D$ and SNR$_D$ denotes the SNR of the downlink.

MUT vs. MUD

- Let $P = W^*$. Then, from (3) and (6), we have

$$P_U = \begin{bmatrix}
|w^*_1 h_1|^2 & \cdots & |w^*_1 h_K|^2 & \sigma^2_U \|w_1\|^2 \\
|w^*_2 h_1|^2 & \cdots & |w^*_2 h_K|^2 & \sigma^2_U \|w_2\|^2 \\
\vdots & \ddots & \vdots & \vdots \\
|w^*_K h_1|^2 & \cdots & |w^*_K h_K|^2 & \sigma^2_U \|w_K\|^2
\end{bmatrix} \left\{ x_1, x_2, \ldots, x_K \right\}$$  

(12)

$$P_D = \begin{bmatrix}
|h^*_1 p_1|^2 & |h^*_1 p_2|^2 & \cdots & |h^*_1 p_K|^2 \\
|h^*_2 p_1|^2 & |h^*_2 p_2|^2 & \cdots & |h^*_2 p_K|^2 \\
\vdots & \ddots & \vdots & \vdots \\
|h^*_K p_1|^2 & |h^*_K p_2|^2 & \cdots & |h^*_K p_K|^2 \\
\sigma^2_D \|p_1\|^2 & \sigma^2_D \|p_2\|^2 & \cdots & \sigma^2_D \|p_K\|^2
\end{bmatrix} \left\{ x_1, x_2, \ldots, x_K \right\}$$  

(13)
MUT vs. MUD

- The total undesired power of the uplink is given by
  \[ P_{UI} = \sum_{k=1}^{K} \sum_{l=1,l\neq k}^{K} |w_k^T h_l|^2 + \sigma_U^2 \sum_{k=1}^{K} \|w_k\|^2 \]  
  \[ (14) \]

- The total undesired power of the downlink is given by
  \[ P_{DI} = \sum_{k=1}^{K} \sum_{l=1,l\neq k}^{K} |h_k^T p_l|^2 + \sigma_D^2 \sum_{k=1}^{K} \|p_k\|^2 \]
  \[ = \sum_{k=1}^{K} \sum_{l=1,l\neq k}^{K} |p_k^T h_l|^2 + \sigma_D^2 \sum_{k=1}^{K} \|p_k\|^2 \]  
  \[ (15) \]

From MUD to MUT - Observations

- An E-optimum solution in MUD is equivalent to an A-optimum solution in MUT; 
- The O-optimum solution, which is optimum with respect to all the users in the system, can be achieved by finding either the E-optimum or A-optimum solutions for all the users involved; 
- For any a given linear MUD algorithm, there exists a counterpart linear MUT algorithm.

MUT vs. MUD - Observations when \( P = W^* \)

- \( P_D = P_U \) and \( P_{DI} = P_{UI} \), if \( \sigma_D^2 = \sigma_U^2 \);
- For any a given item appearing in (12), it appears in (13) once and only once in the form of \( |(w_k^T h_l)|^2 \);
- When the item \( |w_k^T h_l|^2 \) in (12) is minimized in some sense, the item \( |h_l^T p_k|^2 \) in (13) is minimized in the same sense. The former denotes the interference of \( x_l \) imposing on the desired symbol \( x_k \), while the later denotes the interference of the desired symbol \( x_k \) imposing on \( x_l \);
- Hence, when a weight vector \( w_k \) is E-optimum for detection, which minimizes the interference imposed on the desired MT \( k \), the corresponding preprocessing vector \( p_k = w_k^* \) is then A-optimum for transmission, which minimizes the interference generated by the symbol transmitted to the desired MT \( k \).

From MUD to MUT

- The O-optimum downlink preprocessing matrix \( P \) can be obtained from the optimum uplink post-processing matrix \( W \) as
  \[ P = \text{Modified} \left( W^* | \rho \sigma_D^2 \rightarrow \sigma_U^2 \right) \beta \]  
  \[ (16) \]
  \[ \beta = \text{diag} \{ \beta_1, \beta_1, \ldots, \beta_K \} \] is for implementing power allocation; 
  \[ a \rightarrow b \] means using \( a \) to replace \( b \); 
  \[ \rho \] is defined as the noise-suppression factor, which can be optimized according to the specific communications environment.
- The A-optimum preprocessing vectors \( \{p_k\} \) can be obtained from the E-optimum vectors \( \{w_k\} \) as
  \[ p_k = \beta_k \times \text{Modified} \left( w_k^* | \rho \sigma_D^2 \rightarrow \sigma_U^2 \right) , \ k = 1, \ldots, K \]  
  \[ (17) \]
MUT vs. MUD: Summary

- Let \( W \) or \( w_k \) \((k = 1, \ldots, K)\) be the post-processing matrix or vectors. Let the post-processing be expressed as

\[
z = W^H y = W^H H x + W^H n_U,
\]

\[
z_k = w_k^H h_k x_k + w_k^H \left( \sum_{l \neq k} h_l x_l + n_U \right), \quad k = 1, 2, \ldots, K
\]

(18)

- Then, once the post-processing matrix \( W \) or vector \( w_k \) is obtained with the aid of an optimization scheme, the preprocessing matrix \( P \) or vectors \( p_k \) \((k = 1, \ldots, K)\) can then be obtained according to

\[
P = \text{Modified} \left( W^* | \sigma^2_D \rightarrow \sigma^2_U \right) \beta
\]

\[
p_k = \beta_k \times \text{Modified} \left( w_k^* | \sigma^2_D \rightarrow \sigma^2_U \right), \quad k = 1, 2, \ldots, K
\]

(19)

MUT vs. MUD: Design Examples

- Matched-filtering;
- Zero-forcing;
- Minimum mean-square error;
- Minimum variance distortionless response;
- Minimum power distortionless response;
- Maximum signal-to-interference-plus-noise ratio.

From MF Single-User Detector to MF Single-User Transmitter

In the context of the MF single-user detector,

- the post-processing matrix \( W \) in (3) is given by

\[
W = H
\]

(20)

- For the MF single-user transmitter, the preprocessing matrix \( P \) in (6) can be expressed as

\[
P_{\text{MF}} = W^* \beta = H^* \beta
\]

(21)

where \( \beta \) is for implementation of power-allocation.

From Zero-Forcing MUD to Zero-Forcing MUT

- The optimum post-processing matrix \( W \) in (3) in ZF sense can be obtained according to

\[
W_{\text{ZF}} = \arg \left\{ W^H H = I \right\}
\]

(22)

which yields

\[
W_{\text{ZF}} = H \left( H^H H \right)^{-1};
\]

(23)

- Correspondingly, the optimum preprocessing matrix \( P \) in the sense of ZF, i.e., the ZF-MUT, can be expressed as

\[
P_{\text{ZF}} = W^* \beta = H^* \left( H^T H^* \right)^{-1} \beta,
\]

(24)

which is the well-known preprocessing matrix given in the literature.
MMSE MUD

- When the MMSE detector is considered, the post-processing matrix \( W \) or vector \( w_k \) is chosen such that the mean-square error (MSE) between the transmitted symbol vector \( x \) or \( x_k \) and its estimate \( \hat{x} \) or \( \hat{x}_k \) in (18) is minimized.
- Hence, for the joint-optimization MMSE (all users considered together), the optimization problem can be formulated as
  \[
  W_{\text{MMSE}} = \arg \min_W E \left[ \| x - \hat{x} \|^2 \right] = \arg \min_W E \left[ \| x - WHx - WHn_U \|^2 \right]
  \]  

- By contrast, for the E-optimization MMSE (consider only one user), the optimization problem can be formulated as
  \[
  w_{\text{MMSE},k} = \arg \min_{w_k} \left[ x_k - w_k^H v_k x_k - w_k^H n_i^{(k)} \right]^2
  \]

  where \( n_i^{(k)} = \sum_{l \neq k} h_l x_l + n_U \).

From MMSE-MUD to MMSE-MUT

- The E-optimum solution in MMSE-MUD described in (26) is given by
  \[
  w_{\text{MMSE},k} = \frac{(R_i^{(k)})^{-1} h_k}{1 + h_k^H (R_i^{(k)})^{-1} h_k}, \quad k = 1, \ldots, K
  \]  

  where \( R_i^{(k)} \) represents the auto-correlation matrix of the MUI plus noise
  \[
  R_i^{(k)} = \sum_{l=1,l \neq k}^K h_l h_l^H + \sigma_U^2 I_N;
  \]

- Correspondingly, the A-optimum solution in MMSE-MUT can be obtained as
  \[
  p_{\text{MMSE},k} = \beta_k \times \text{Modified} \left( w_k^* | \rho \sigma_D^2 \to \sigma_U^2 \right)
  \]

  where \( \beta_k \) is chosen such that the mean-square error (MSE) between the transmitted symbol vector \( x \) or \( x_k \) and its estimate \( \hat{x} \) or \( \hat{x}_k \) in (18) is minimized.

  

MMSE-MUT vs. MMSE-MUD: Physical Principles

- The post-processing vector \( w_{\text{MMSE},k} \) of (29) minimizes the MSE in the context of the MUI of the \((K - 1)\) interfering users imposing on the desired user \( k \) and of the background noise observed at the desired (uplink) user \( k \);

- The preprocessing vector \( p_{\text{MMSE},k} \) of (30) with an optimum \( \rho \) factor minimizes the MSE in terms of the interference of the desired user \( k \) imposing on its \((K - 1)\) colleague users and of the background noise observed at the desired (downlink) user \( k \).
From MVDR-MUD to MVDR-MUT

- For the joint-optimization MVDR, there is no interference and the variance of \( z \) is given by
  \[
  \text{Var}(z) = E[\|Wn_u\|^2] = \sigma_u^2 \text{Trace}(W^H W)
  \]  
  (31)

- The criterion of distortionless response can be expressed as
  \[
  W^H H = I_K
  \]  
  (32)

- The optimization problem can be formed as
  \[
  W_{\text{MVDR}} = \arg \min_W \{\sigma_u^2 \text{Trace}(W^H W)\}
  \]
  subject to \( W^H H = I_K \)
  (33)

- Upon taking the complex gradient of Trace(\( J \)) with respect to \( W^* \) and equating it to zero, it gives
  \[
  W_{\text{MVDR}} = \sigma_u^2 H \lambda
  \]  
  (35)

- Substituting (35) into (32) yielding
  \[
  \lambda = \sigma_u^2 (H^H H)^{-1}
  \]  
  (36)

- Finally, when substituting the above equation into (35), we have the post-processing matrix
  \[
  W_{\text{MVDR}} = H (H^H H)^{-1}
  \]  
  (37)

which is the same as the post-processing matrix for the ZF-MUD seen in (23).

From MVDR-MUD to MVDR-MUT

- Therefore, for the joint-optimization MVDR-MUT, the transmitter preprocessing matrix can be formed as
  \[
  P_{\text{MVDR}} = W^* \beta^k
  \]
  \[
  = H^* (H^T H^*)^{-1} \beta^k
  \]  
  (38)

which is the same as the pre-processing matrix for the ZF-MUT seen in (24).

From MVDR-MUD to MVDR-MUT

- In the context of the E-optimization assisted MVDR-MUD, the optimization problem can be formed as
  \[
  w_{\text{MVDR}, k} = \arg \min_{w_k} \{w_k^H R_i^{(k)} w_k\}, \text{ subject to } w_k^H h_k = 1
  \]  
  (39)

- With the aid of the Lagrange multiplier, it can be shown that the optimum solution is
  \[
  w_{\text{MVDR}, k} = \frac{(R_i^{(k)})^{-1} h_k}{h_k^H (R_i^{(k)})^{-1} h_k}
  \]  
  (40)

- Hence, the A-optimum assisted preprocessing vector \( p_{\text{MVDR}, k} \) for the \( k \)th downlink user is
  \[
  p_{\text{MVDR}, k} = \beta_k (R_i^{(k)})^{-1} h_k^*, \quad k = 1, 2, \ldots, K
  \]  
  (41)

which is the same solution as for the A-optimum MMSE-MUT.
MVDR-MUT vs. MVDR-MUD: Physical Principles

- The MVDR-MUD having the post-processing vector of (40) minimizes the variance of the background noise presented at user $k$ plus the interference imposed by the $(K-1)$ uplink interfering users on the desired user $k$, while satisfying the distortionless condition of (32);
- Therefore, the post-processing vector of (40) is E-optimum in MVDR sense;
- By contrast, the MVDR-MUT having the preprocessing vector of (41) minimizes the variance of the background noise presented at user $k$ plus the interference that the desired user $k$ imposes on the other $(K-1)$ downlink users;
- Correspondingly, the preprocessing vector of (41) is A-optimum in MVDR sense.

From MPDR-MUD to MPDR-MUT

- Correspondingly, for the joint MPDR-MUT, the preprocessing matrix $P$ can be expressed as
  \[ P_{\text{MPDR}} = \hat{R}_y^{-1} H^* \left( H^T \hat{R}_y^{-1} H^* \right)^{-1} \beta \]  
  (45)
  associated with $\hat{R}_y = H^* H^T + \rho \sigma_D^2 I_N$.
- Hence, we have
  \[ P_{\text{MPDR}} = \left( H^* H^T + \rho \sigma_D^2 I_N \right)^{-1} H^* \left( H^T \left( H^* H^T + \rho \sigma_D^2 I_N \right)^{-1} H^* \right)^{-1} \beta \]  
  (46)
- Furthermore, using the matrix inverse lemma, it can be simplified to
  \[ P_{\text{MPDR}} = H^* \left( H^T H^* \right)^{-1} \beta \]  
  (47)
  which is also the ZF-MUT solution.

From MPDR-MUD to MPDR-MUT

- For the joint optimization assisted MPDR-MUD, the output power after the linear post-processing is given by
  \[ P_{\text{power}} = \mathbb{E} \left[ \| W^H y \|^2 \right] = \text{Trace} \left( W^H R_y W \right) \]  
  (42)
  where $R_y$ is the auto-correlation matrix of the received signal $y_U$;
- With the distortionless condition as shown in (32), the cost-function for the MPDR-MUD can be written as
  \[ J = W^H R_y W - (W^H H - I_K) \lambda - \lambda^H (H^H W - I_K) \]  
  (43)
- Upon solving the above optimization problem, the optimum post-processing matrix for the MPDR-MUD can be expressed as
  \[ W_{\text{MPDR}} = R_y^{-1} H \left( H^H R_y^{-1} H \right)^{-1} \]  
  (44)
- In the context of the E-optimization MPDR-MUD, the optimization problem can be formed as
  \[ w_{\text{MPDR}, k} = \arg \min_{w_k} \{ w_k^H R_y^{(k)} w_k \}, \text{ subject to } w_k^H h_k = 1 \]  
  (48)
- The solution of (48) is
  \[ w_{\text{MPDR}, k} = \frac{R_y^{-1} h_k}{h_k^H R_y^{-1} h_k} \]  
  (49)
- Therefore, the A-optimum preprocessing vector $p_{\text{MPDR}, k}$ for the $k$th downlink user is
  \[ p_{\text{MPDR}, k} = \beta_p \hat{R}_y^{-1} h_k, \quad k = 1, 2, \ldots, K \]  
  (50)
  where $\hat{R}_y = H^* H^T + \rho \sigma_D^2 I_N$. 
MPDR-MUT vs. MPDR-MUD: Physical Principles

- The MPDR-MUD having the post-processing vector of (49) minimizes the output power of the $k$th user’s detector, while satisfying the distortionless constraint of (32);
- Therefore, the post-processing vector of (49) is E-optimum in MPDR sense;
- By contrast, the MPDR-MUT having the preprocessing vector of (50) minimizes the power in terms of the $k$th user, while satisfying the distortionless constraint of (32);
- In principle, while the distortionless condition is satisfied, minimizing the transmitted power in terms of the $k$th user results in the minimization of the power interfering the other downlink users;
- Hence, the preprocessing vector of (50) is A-optimum in MPDR sense.

From MSINR-MUD to MSINR-MUT

- For the maximum signal-to-interference-plus-noise-ratio MUD (MSINR-MUD), the E-optimum post-processing vector $w_{\text{MSINR},k}$ for the $k$th user is chosen according to
  \[ w_{\text{MSINR},k} = \arg \max_{w_k} \left\{ \text{SINR} = \frac{||w_k^H h_k||^2}{w_k^H R_i^{(k)} w_k} \right\} \]  

- It can be readily shown that
  \[ w_{\text{MSINR},k} = \mu_k \left( R_i^{(k)} \right)^{-1} h_k, \quad k = 1, 2, \ldots, K \]  

where $\mu_k > 0$ is a constant.
- Hence, the A-optimum preprocessing vector for the MSINR-MUT can be converted from (52) as
  \[ p_{\text{MSINR},k} = \beta_k \left( R_i^{(k)} \right)^{-1} h_k^*, \quad k = 1, 2, \ldots, K \]  

which is also the same as the MMSE-MUT solution.

MSINR-MUT vs. MSINR-MUD: Physical Principles

- From (12) and the MSINR principles, it can be shown that applying (52) for the MUD results in the maximization of the ratio between the desired user’s signal power and the power of noise plus the interference imposed by the $(K-1)$ interfering users on the desired user;
- Hence, $w_{\text{MSINR},k}$ is E-optimum in MSINR sense for uplink detection;
- By contrast, from (13) and also the MSINR principles, applying (53) for the MUT results in the maximization of the ratio between the desired user’s signal power and the power of its noise plus the interference of the desired user imposing on the other $(K-1)$ users;
- Therefore, $p_{\text{MSINR},k}$ is A-optimum in MSINR sense for downlink transmission.

Joint Base-Station MUD and MUT in TDD-Mode

- In TDD-mode the uplink multiple-access channels are reciprocal with the downlink broadcast channels;
- Hence, in TDD-based MIMO systems, when the downlink SNR value is not significantly different from the uplink SNR value, the post-processing matrices, say $W$ or $w_k$, $k = 1, 2, \ldots, K$, for the uplink MUD may be directly converted to the preprocessing matrix (vectors), $P$ or $p_k$, $k = 1, 2, \ldots, K$ for the downlink MUT;
- However, for the downlink preprocessing, the power allocation should be considered.
Joint Base-Station MUD and MUT in TDD-Mode (continued)

- Specifically, the transmitter preprocessing matrix $P$ or vectors $p_k$ for $k = 1, 2, \ldots, K$ can be formed from the receiver post-processing matrix $W$ or vectors $w_k$ for $k = 1, 2, \ldots, K$ according to

$$P = W^*\beta$$
$$p_k = \frac{1}{\sqrt{\|w_k\|^2}}w_k^*, \quad k = 1, 2, \ldots, K$$

(54)

- Therefore, we are indicated that nearly any of the adaptive MUD algorithms in the literature, such as the adaptive decorrelating, adaptive MMSE, adaptive MSINR, adaptive minimum BER (MBER), etc. MUDs and the adaptive reduced-rank MUDs may be invoked not only for implementing the uplink detection,

- but also for providing the downlink transmitter preprocessing at the same time.

Joint Base-Station MUD and MUT in TDD-Mode (continued)

- In practice this technique can be applied, provided that the difference between the uplink and downlink SNR values is within a certain range;

- By doing this, a lot of implementation complexity may be saved and the transceivers may be implemented with extremely low-complexity, since in this case both the uplink (incoming) detection and the downlink transmitter (outgoing) preprocessing share the same adaptive signal processing algorithm.

Joint Adaptive MUD and MUT

- Let at the BS the uplink detected signal and downlink transmission signal be expressed as

$$z = W^H(n)y(n), \quad z_k(n) = w_k^H(n)y(n), \quad k = 1, 2, \ldots, K$$

(55)

$$d(n) = P(n)x(n) = \sum_{k=1}^{K} p_k(n)x_k(n)$$

(56)

- Let the BS forms the $(n+1)$th post-processing matrix or vectors in (55) according to a certain adaptive filtering algorithm as

$$W(n+1) = f[W(n), W(n-1), \cdots |\text{parameters}]$$

$$w_k(n+1) = f[w_k(n), w_k(n-1), \cdots |\text{parameters}], \quad k = 1, 2, \ldots, K$$

(57)

(58)

- Based on (57) or (58), the transmitter preprocessing matrix or vectors in (58) at time $(n+1)$ can be simply formed as

$$P(n+1) = W^*(n+1)\beta(n+1)$$

(59)

$$p_k(n+1) = \beta_k w_k^*(n+1), \quad k = 1, 2, \ldots, K$$

(60)
Examples of Performance Results

- A downlink space-division multiple-access (SDMA) system;
- \( N \): Transmit antennas at the BS;
- One receive antenna per MT;
- Binary phase-shift keying (BPSK) modulation;
- Independent frequency non-selective (flat) Rayleigh fading channels.

Figure 3: Schematic block diagram of joint adaptive multiuser detector and multiuser transmitter preprocessor.

Figure 4: **MF**: BER versus SNR per bit performance for the downlink SDMA system, when communicating over flat Rayleigh fading channels.

Figure 5: **Zero-forcing**: BER versus SNR per bit performance for the downlink SDMA system, when communicating over flat Rayleigh fading channels.
Figure 6: **MMSE**: BER versus SNR per bit performance for the downlink SDMA system, when communicating over flat Rayleigh fading channels.

Figure 7: **MMSE**: BER versus the noise-suppression factor, $\rho$, performance for the downlink SDMA system using $N = 10$ transmit antennas and each mobile terminal using one receive antenna, when communicating over flat Rayleigh fading channels.

Figure 8: **MMSE**: BER versus the noise-suppression factor, $\rho$, performance for the downlink SDMA system using $N = 10$ transmit antennas and each mobile terminal using one receive antenna, when communicating over flat Rayleigh fading channels.

### Summary

- For any a given linear MUD algorithm, there exists a counterpart linear MUT algorithm: an E-optimum solution in MUD can always be modified to an A-optimum solution in MUT;

- The O-optimum downlink preprocessing matrix $P$ can be obtained from the optimum uplink post-processing matrix $W$ as

  $$P = \text{Modified} \left( W^* \left| \frac{\rho \sigma^2_D}{\sigma^2_U} \rightarrow \sigma^2_U \right. \right) \beta$$

  (61)

- An A-optimum preprocessing vector $p_k$ can be obtained from a corresponding E-optimum vector $w_k$ as

  $$p_k = \beta_k \times \text{Modified} \left( w^*_k \left| \frac{\rho \sigma^2_D}{\sigma^2_U} \rightarrow \sigma^2_U \right. \right) , \quad k = 1, \ldots, K$$

  (62)
Summary (continued)

- In MMSE-type MUT the achievable performance is only loosely dependent on the noise-suppression factor, or the knowledge about the background noise;
- In TDD-based MIMO systems the MUT preprocessing vectors for the downlink transmission can be readily extracted from the MUD post-processing vectors for the uplink detection;
- The weight vectors for both the uplink MUD and downlink MUT may be obtained based on one adaptive algorithm, yielding low-complexity transceivers in TDD-assisted MIMO wireless communications.

Altruistic/Egocentric Optimization: Other Applications

- Any scenarios where co-channel interference exists;
- Wireless ad-hoc and wireless sensor networks (WSNs);
- Cooperative wireless networks;
- Downlink transmission in cellular wireless communications systems: using A-optimization to suppress the intra-cell interference and E-optimization to mitigate inter-cell interference.
- Anything else?

Operations of Fig. 9

- Objectives:
  - to send $x_1$ to $T_1$, $x_2$ to $T_2$, and $x_3$ to $T_3$;
  - to attain two-order of diversity for $x_1$, $x_2$, and $x_3$.
- Operation steps:
  1. $S_1$ sends $x_1$ to $R_1$ and $[x_1, x_2]$ to $R_2$ using A-optimization, and $S_2$ sends $x_3$ to $R_3$ and $[x_2, x_3]$ to $R_2$ using A-optimization;
  2. $R_1$ detects $x_1$ using E-optimization to suppress the interference from $S_2$, $R_2$ detects $[x_1, x_2, x_3]$ using E-optimization to suppress the interference among $[x_1, x_2, x_3]$, and $R_3$ detects $x_3$ using E-optimization to suppress the interference from $S_1$;
- $x_2$ attains two-order of diversity, since it is received from both $S_1$ and $S_2$.  

Figure 9: Application of A/E-optimization in wireless cooperative networks.
Operations of Fig. 9

- Operation steps:
  3. $R_1$ broadcasts $x_1$, $R_2$ broadcasts $[x_1, x_2, x_3]$ using A-optimization, and $R_3$ broadcasts $x_3$;
  4. $T_1$ recovers $x_1$ using E-optimization to suppress the interference from $R_3$, $T_2$ recovers $x_2$ using E-optimization to suppress the interference from both $R_1$ and $R_3$, and $T_3$ recovers $x_3$ using E-optimization to suppress the interference from $R_1$;

- Explicitly, both $x_1$ and $x_3$ are capable of attaining two-order of diversity, since $x_1$ is received from $R_1$ and $R_2$, while $x_3$ is received from $R_2$ and $R_3$.

- Therefore, $x_1$, $x_2$ and $x_3$ all achieve two-order of diversity.

Thank you!