Advanced Low-Complexity Near-ML MUD for SDM/SDMA OFDM Systems
—Sphere Detection

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Outline of the Presentation

- Introduction to Sphere Detection
- Work Plan & Progress
- Research Highlights
  1. The Center-Shifting Theory for SD.
  2. Iterative Center-Shifting SD Aided Receiver Design.
  3. Simulation Results
- Conclusions.
MIMO Detection Classification

MIMO Detection

Linear Detection
- LS
- MMSE

Ordered Successive Interference Cancellation
- V-BLAST

Low-Complexity
- ML

Near-ML
- SD

Depth-First SD
- Conventional SD

Breadth-First SD
- OHRSA
- K-Best SD
BER Performance and Complexity of Classic ML Detector

(a) BER Performance.

(b) Computational Complexity.

Assume: each user has single Tx antenna
Introduction to Sphere Detection

- **ML Metric Equation Transformation**

\[
\hat{s}_{ML} = \arg \min_{\tilde{s} \in \mathcal{M}^c} ||y - H\tilde{s}||^2
\]

\[
= \arg \min_{\tilde{s} \in \mathcal{M}^c} \{(\tilde{s} - \hat{x})^HH^H\mathbf{H}(\tilde{s} - \hat{x}) + y^H(I - H(H^HH)^{-1}H^H)y\},
\]

where \(\hat{x} = (H^HH)^H y\) is the unconstrained ML solution.

- **Channel Triangularization**

\[
(\tilde{s} - \hat{x})^HH^H\mathbf{H}(\tilde{s} - \hat{x}) = (\tilde{s} - \hat{x})^HU^H\mathbf{U}(\tilde{s} - \hat{x}),
\]

\[
= \sum_{i=1}^{M} u_{ii}^2 [\tilde{s}_i - \hat{x}_i + \sum_{j=i+1}^{M} \frac{u_{ij}}{u_{ii}} (\tilde{s}_j - \hat{x}_j)]^2
\]
Introduction to Sphere Detection

- **Partial Euclidean Distance Accumulation**

Hence, we can calculate the accumulated Partial Euclidean Distance (PED) for each \( \tilde{s}_i \) recursively, if we start from \( i = M \). Specifically, for \( i = M \), we have:

\[
PED_M(s_M) = u_{MM}^2 (\tilde{s}_M - \hat{x}_M)^2.
\]  

(5)

Then we can choose a candidate arbitrarily for the \( M \) th Tx antenna. Subsequently, we calculate the PEDs for the \( (M - 1) \) th Tx antenna based on the choice of the \( M \) th Tx antenna. Thus, we have:

\[
PED_{M-1}(\tilde{s}_M) = u_{MM}^2 (\tilde{s}_M - \hat{x}_M)^2
\]

\[
+ u_{M-1,M-1}^2 [\tilde{s}_{M-1} - \hat{x}_{M-1} + \frac{u_{M-1,M}}{u_{M-1,M-1}} (\tilde{s}_M - \hat{x}_M)]^2
\]

\[
= PED_M(\tilde{s}_M) + \phi_{M-1}(\tilde{s}_{M-1}),
\]  

(6)

where \( \phi_i(\tilde{s}_i) = u_{ii}^2 [\tilde{s}_i - \hat{x}_i + \frac{u_{i+1,1}}{u_{i,i}} (\tilde{s}_{i+1} - \hat{x}_{i+1})]^2 \).

Finally, we calculate the accumulated PED for the \( \tilde{s}_i \) by invoking the following recursive formula:

\[
PED_i(\tilde{s}_i) = PED_{i+1}(\tilde{s}_{i+1}) + \phi_i(\tilde{s}_i).
\]  

(7)
Introduction to Sphere Detection

- **Search Restriction**
  Conventionally, two types of search restriction are employed, namely the search radius $C$ and the number of best candidates retained for each dimensional search $K$, which result in following two types of SD, respectively.

1. Depth-First SD
   (a) The depth-first algorithm searches for the potential ML solution in both upward and downward directions along a search tree.
   (b) The computational complexity may be increasingly reduced by shrinking the radius $C$, whenever a leaf is reached.

2. Bread-First SD ($K$-Best SD)
   (a) It searches for the ML candidates in the downward direction only along the search tree.
   (b) The initial search radius is calculated based on LS criterion, namely, we have $C = \| \mathbf{r} - \hat{\mathbf{r}}_{LS} \|$, which is not changed during the search.
   (c) The K Best candidates are retained at each level of the search tree.
   (d) A constant decoding complexity/throughput can be guaranteed.
   (e) However, the K-Best SD does not necessarily find the ML solution.
   (f) There is always a tradeoff between performance and complexity.
Geometrical Representation of the Sphere Detection Assisted by Search Radius $C$

Figure 1: One Dimensional Case
Illustration of $K$-Best SD Algorithm

Figure 2: 4 Dimensional Case
Comparison of Sphere Detectors

- \((4 \times 4)\) 16QAM SDM/OFDM
- Both Depth-First SD and K-Best SD can achieve ML BER performance with much lower complexity.
- Depth-First SD: SNR-sensitive complexity.
- K-Best SD: SNR-independent complexity.

<table>
<thead>
<tr>
<th></th>
<th>Depth-First SD</th>
<th>K-Best SD (K=16)</th>
<th>OHRSA</th>
</tr>
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<tbody>
<tr>
<td>Initial Search Radius</td>
<td>Required</td>
<td>Optional</td>
<td>Not Required</td>
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<tr>
<td>Memory Requirement</td>
<td>Moderate</td>
<td>Relatively High</td>
<td>Moderate</td>
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<tr>
<td>BER Performance</td>
<td>ML</td>
<td>ML (not guaranteed)</td>
<td>ML</td>
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<tr>
<td>SNR-Sensitive Complexity</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Average Complexity</td>
<td>Polynomial (moderate rates)</td>
<td>Determined by the value of K</td>
<td>Same as Depth-First SD</td>
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<tr>
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<td>Exponential (high rates)</td>
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<td>Tree Search Direction</td>
<td>two-direction</td>
<td>single-direction</td>
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<tr>
<td>Throughput</td>
<td>Variable</td>
<td>Constant</td>
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<td>Hardware Implementations</td>
<td>Demand I/O buffers</td>
<td>Easy in a pipeline architecture</td>
<td>Demand I/O buffers</td>
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Research Outline & Progress

• Work Plan
  1. **Comparative Study**: Various sphere detectors (SD).
  2. **Computational Complexity Reduction**:
     (a) Investigation on different complexity-reduction schemes.
     (b) Investigation on iterative center-shifting SD.
     (c) Investigation on the *a priori*-LLR-Threshold aided SD.
  3. **Throughput Improvement**: Advanced K-Best SD aided 3-stage iterative receiver architecture.
  4. **Trade-off between Capacity and Complexity**:
     (a) Investigation on the multi-functional MIMO system, where the above-mentioned advanced turbo receiver is expected to be employed.
     (b) Investigation on hybrid/irregular sphere detector.

• Progress
  1. **Comparative Study** (Done)
  2. **Complexity Reduction**: Center-shifting based iterative SD detection
     (a) EXIT-Chart-Aided Direct Search Center Feedback. (Done)
     (b) MMSE based Search Center Shifting. (in Progress)
  3. **Throughput Improvement**: 3-Stage iterative receiver
     (a) Add Unit-Rate-Code (URC) precoder and its corresponding decoder blocks to the original system. (Done)
     (b) Investigate center-shifting SD aided 3-stage iterative receiver. (in Progress)
Research Highlight—The Center-Shifting Theory for SDs

Recall that the well-known ML solution of $\hat{s}_{ML} = \arg\min_{\hat{s} \in M_{c}^{M}} ||y - H\hat{s}||^{2}$ can be obtained by solving

$$\hat{s}_{ML} = \arg\min_{\hat{s} \in M_{c}^{M}} (\hat{s} - \hat{x})^{H}H^{H}H(\hat{s} - \hat{x}),$$

where $\hat{x} = (H^{H}H)^{H}y$. Although we found that setting the SD's search center $\hat{x}$ to an arbitrary point does not necessarily yield the ML performance, for example, when $\hat{x}$ is worse than the LS solution, a better BER performance is expected as the search center $\hat{x}$ approaches to the ML solution. Hence, instead of fixing the search center conventionally, we could update the search center to a point closer to the real ML solution during the iterative detection process, as portrayed below:
Research Highlight—The Iterative Center-Shifting SD Aided Receiver Design

We have proposed three receiver architecture employing the center-shifting SD:

Direct-Hard-Decision Center-Shifting (DHDC):

- Imposing hard decision directly on a posteriori LLR at the output of the channel decoder.
- Discarding all soft information contained in the a posteriori LLRs.

Direct-Soft-Decision Center-Shifting (DSDC):

- Calculate the soft-symbol based on the a posteriori LLR at the output of the channel decoder.
- Retains the soft-information contained in the a posteriori LLRs.
Research Highlight—The Iterative Center-Shifting SD Aided Receiver Design

MMSE Soft-Interference-Cancellation (SIC-MMSE):

- Calculating the search center by invoking the SIC-MMSE scheme on the a priori LLRs gleaned from the input of the SD.
- Slightly increased complexity imposed by the SIC-MMSE, but significant performance improvement with the aid of the center-shifting scheme.
EXIT Chart Analysis

(a) 3D EXIT Chart

(b) 2D EXIT Chart (Projection)

Figure 3: DHDC-Aided SD in $(8 \times 4)$-Element 4-QAM SDM/OFDM System
EXIT Chart Analysis

(a) 3D EXIT Chart Comparison of DHDC and DSDC Schemes

(b) EXIT Chart of SIC-MMSE Center-Shifting Scheme

Figure 4: (8 × 4)-Element 4-QAM SDM/OFDM System
Simulation Results—Performance

(a) Performance Gain Achieved by the SIC-MMSE Center-Shifting Scheme

(b) BER Performance Comparison of the DHDC, the DSDC, and the SIC-MMSE Center-Shifting Schemes
Simulation Results—Complexity

- The detection-candidate-list-generation-related complexity was reduced by an order of magnitude.
- The extrinsic-LLR-calculation-related complexity was reduced by a factor of 64.
- The associated memory requirements were also reduced by an order of magnitude.

Performance Gain & Computational Complexity Reduction Achieved by the SIC-MMSE Scheme in an \((8 \times 4)\)-element 4-QAM SDM/OFDM Rank-Deficient System

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<th>BER</th>
<th>Center-Shifting</th>
<th>(N_{\text{can}}(K))</th>
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<th>Memories</th>
<th>SD Compl.</th>
<th>MAP Compl.</th>
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Thank You For Your Attention!